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2001 J. Phys.: Condens. Matter 13 L147

(http://iopscience.iop.org/0953-8984/13/6/103)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) L147–L152 www.iop.org/Journals/cm PII: S0953-8984(01)18513-9

LETTER TO THE EDITOR

Transport across a normal–superconducting interface: a novel probe of electron–electron interactions in the normal metal

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Received 31 October 2000

Abstract

In this letter, we examine the effect of Coulomb interactions in the normal region of a normal–superconducting (N/S) mesoscopic structure; here the change from an attractive to a repulsive Coulombic interaction, at the N/S interface, causes a shift in the order parameter phase. We show that this shift has a pronounced effect on the Andreev bound states and demonstrate that the effect on Andreev scattering of non-zero order parameter tails can be used to probe the sign of the interaction in the normal region.

Recent advances in materials technology have enabled the fabrication of normal/superconducting (N/S) mesoscopic hybrid structures with well defined dimensions and interfaces [1–4]. Due to the proximity of the normal material to the superconductor, the pairing field $f(x) = \langle\psi_\uparrow(x)\psi_\downarrow(x)\rangle$, in the normal region, decays to zero on the scale of a coherence length ξ [5]. During the past decade this proximity effect has been extensively investigated both experimentally and theoretically (see for example [6–9]).

In contrast to the pairing field $f(x)$, the effective electron–electron interaction, $V(x)$, changes abruptly at the S/N interface, from an attractive interaction in the superconductor to either zero, a much diminished attractive interaction or a repulsive interaction, in the normal (N) material. Consequently the order parameter, $\Delta = V(x)f(x)$, of an s-wave superconductor also changes abruptly at the S/N interface, as shown in figure 1. To date, apart from a small number of notable exceptions [10], theoretical research into the transport properties of N/S interfaces has mainly considered the order parameter in the normal region to be zero (figure 1(a)) [11,12].

In this letter we consider the effect of an attractive or repulsive electron–electron interaction ($V(x) \neq 0$), as shown in figures 1(b) and 1(c). The difference between figures 1(b) and 1(c) is the π phase shift in $\Delta(x)$, induced by the crossover from an attractive to a repulsive interaction at the N/S interface. In what follows we examine how this phase shift affects the transport properties associated with Andreev bound states.

To investigate this regime we adopt a general scattering approach to dc transport, which was initially developed to describe phase-coherent transport in dirty mesoscopic superconductors [13]. For simplicity, in this letter, we focus solely on the zero-voltage, zero-temperature conductance, for the structure shown in figure 2. In the linear-response limit, at

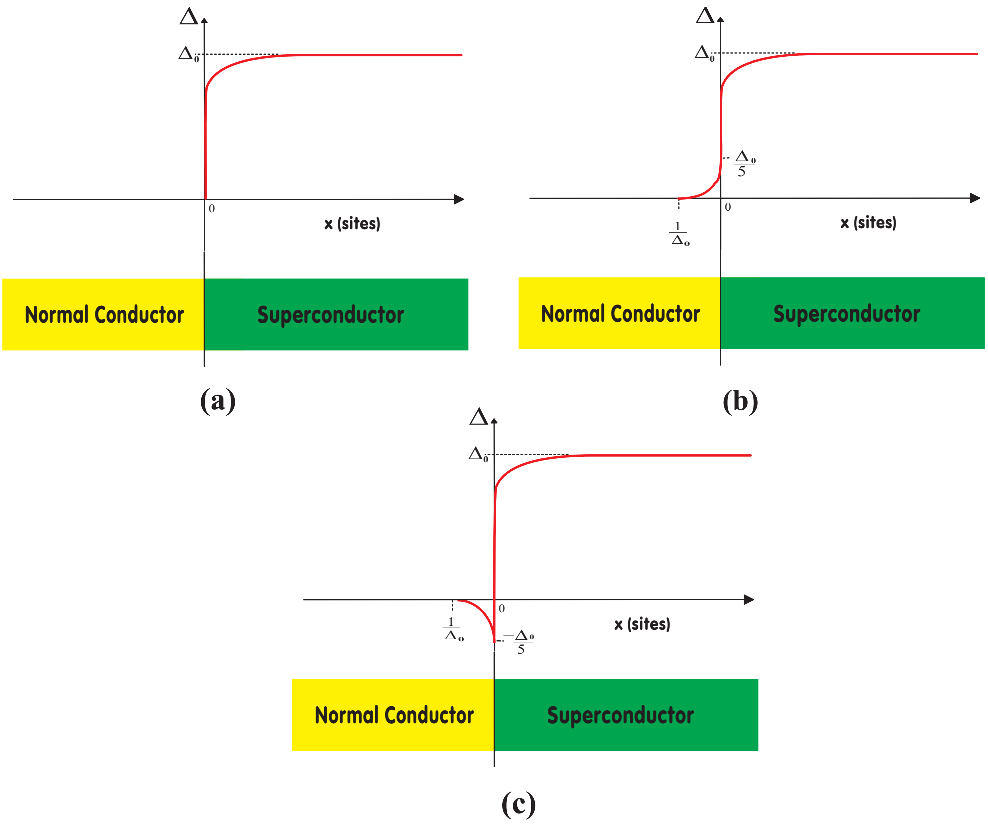

Figure 1. (a) The self-consistent order parameter profile; $V(x) = 0$ in the normal region. (b) The self-consistent order parameter profile; $V(x)$ and $f(x)$ are both finite in the normal region. (c) The self-consistent order parameter profile, including Coulomb interactions.

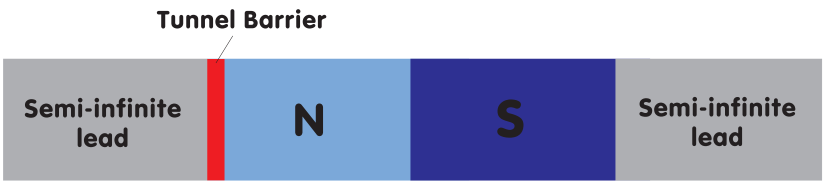

Figure 2. A N–S system with a tunnel barrier present. Such a system will have Andreev bound states present in the region between the interface and the tunnel barrier.

zero temperature, the conductance of a phase-coherent structure may be calculated from the fundamental current–voltage relationship [14, 15]:

$$I_i = \sum_{j=1}^{2} a_{ij}(v_j - v). \tag{1}$$

The above expression relates the current I_i from a normal reservoir i to the voltage differences $(v_j - v)$, where $v = \mu/e$ and the sum is over the two normal leads connected to the scattering region. The a_{ij} are linear combinations of the coefficients for normal and Andreev scattering and in the absence of superconductivity satisfy $\sum_{j=1}^{2} a_{ij} = \sum_{i=1}^{2} a_{ij} = 0$ in which case the left-hand side of equation (1) becomes independent of v. In units of $2e^2/h$ [14, 15], $a_{ii} = N_i + R_i^A - R_i^0$ and $a_{ij\neq i} = T_{ij}^A - T_{ij}^0$, where T_{ij}^A, T_{ij}^0 are Andreev and normal coefficients of transmission from probe j to probe i, R_i^A, R_i^0 are Andreev and normal coefficients of reflection from probe i and N_i is the number of open scattering channels in lead i.

Setting $I_1 = -I_2 = I$ and solving equation (1) for the two-probe conductance yields [16]

$$G = \frac{I}{(V_1 - V_2)} = T_{21}^0 + T_{12}^A + \frac{2(R_2^A R_1^A - T_{21}^A T_{12}^A)}{R_2^A + R_1^A + T_{21}^A + T_{12}^A} \tag{2}$$

where G is the conductance in units of $2e^2/h$. As noted in [16], the various transmission and reflection coefficients can be computed by solving the Bogoliubov–de Gennes equation on a tight-binding lattice of sites, where each site is labelled by an index i and possesses a particle (hole) degree of freedom $\psi(i)$ ($\phi(i)$). In the presence of local s-wave pairing described by a superconducting order parameter Δ_i, this takes the form

$$\begin{aligned} E\psi_i &= \epsilon_i\psi_i - \sum_\delta \gamma(\psi_{i+\delta} + \psi_{i-\delta}) + \Delta_i\phi_i \\ E\phi_i &= -\epsilon_i\phi_i + \sum_\delta \gamma(\phi_{i+\delta} + \phi_{i-\delta}) + \Delta_i^*\psi_i. \end{aligned} \tag{3}$$

In what follows, the Hamiltonian of equation (3) is used to describe the structure of figure 2, where for all i, the on-site energy $\epsilon_i = \epsilon_0$. In the S region, the order parameter Δ_i is set to a constant, $\Delta_i = \Delta_0$, while in the normal region Δ_j is approximated by

$$\Delta_j = \pm\frac{\Delta_0}{5}(\tanh(j - L_n) + 1). \tag{4}$$

The nearest-neighbour hopping element γ merely fixes the energy scale (i.e. the bandwidth), whereas ϵ_0 determines the band filling and L_n is the length of the normal region. In what follows we choose $\gamma = 1$ and $\Delta_0 = 0.1$. By numerically solving for the scattering matrix of equation (3), exact results for the dc conductance can be obtained and therefore the effects of a repulsive/attractive Coulomb interaction in the normal region can be examined.

For simplicity, in this letter, we consider the transport properties of the structure shown in figure 2. It consists of two normal, semi-infinite, crystalline leads, 20 sites wide, joined by a scattering region. The normal-scattering region is 40 sites long and the superconducting region is of length L_S. To form Andreev bound states we create quasiparticle confinement in the area in front of the superconductor by introducing a weak point-like contact between the left lead and the scattering region, and ensuring that there is no or very little quasiparticle transmission through the superconductor. For this reason the superconductor length was set to $L_S = 150$ sites. However, care must be taken since the decay length of the sub-gap states in the superconductor increases with quasiparticle energy. At energies close to the gap energy the decay length is long enough for transmission through the superconductor; see figure 3. The weak contact is created by placing a potential barrier, one site wide, between the left lead and the normal-scattering region, via a mean potential U added to the on-site energy ϵ_0. Figure 4 shows a plot of the transmission coefficient of the barrier as a function of U. From this plot we see that for a low quasiparticle transmission through the barrier, a high potential is needed. For these reasons the barrier potential was set at $U = 20$.

These conditions produce discrete states within the normal regions. Allowing the formation of Andreev bound states, when these energy levels become populated resonances appear in the conductance of the system, analogous to Breit–Wigner resonances [17]. By plotting G as a function of quasiparticle energy (all energies are with respect to the Fermi energy) we are able to investigate transport resonances in the conductance due to the formation of Andreev bound states. Figure 5 shows the conductance as a function of quasiparticle energy for the

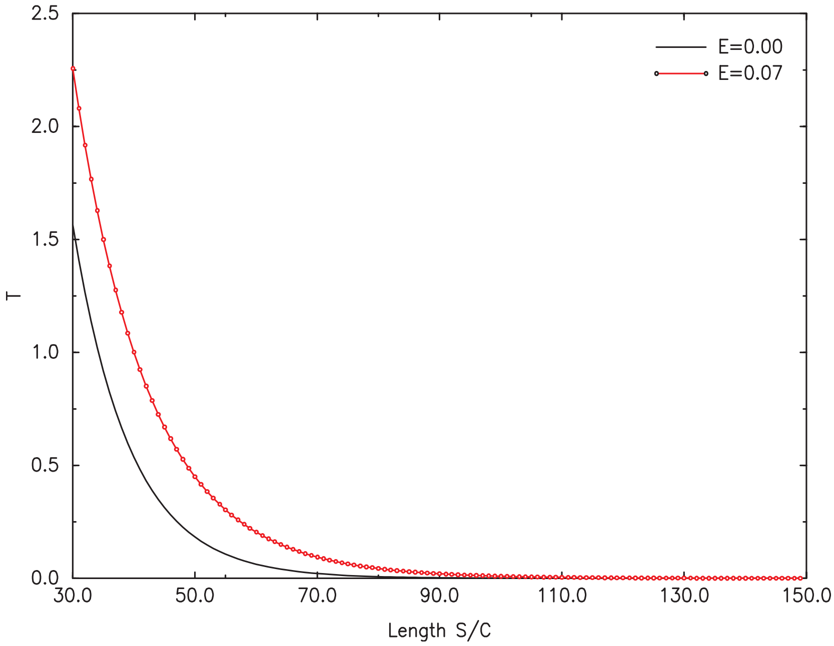

Figure 3. A graph showing the transmission coefficient as a function of the superconductor length for various quasiparticle energies. No barrier or proximity tail is present. The structure width is 20 sites. In this graph it can be seen that at high quasiparticle energies, transmission will occur through the superconductor.

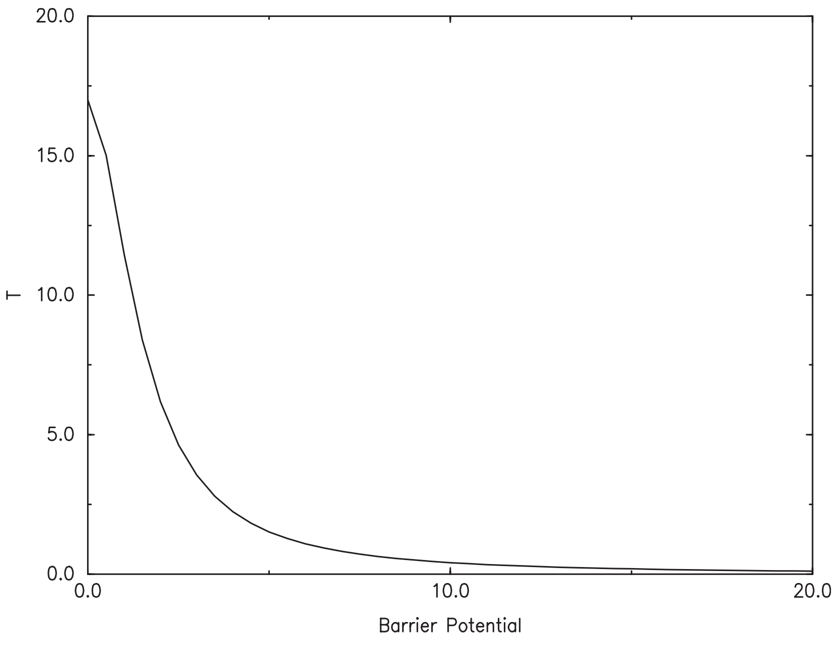

Figure 4. A graph showing the effect of the barrier potential. The barrier is one site wide; the structure width is 20 sites. The graph shows transmission as a function of barrier potential for a quasiparticle energy $E = 0.00$.

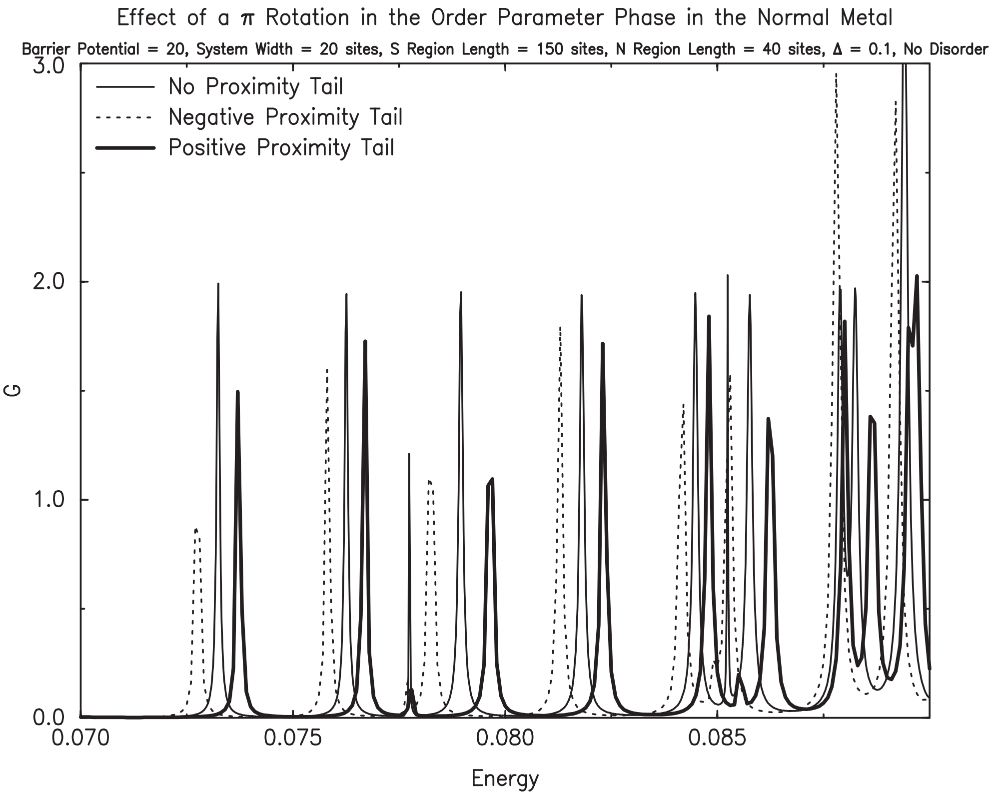

Figure 5. Graphs of conductance as a function of quasiparticle energy for different phases of the order parameter tail in the normal metal.

three proximity tails corresponding to the vanishing, attractive and repulsive electron–electron interactions. These represent the central result of this letter. We see that the introduction of a proximity tail has the effect of shifting the energy at which the Andreev bound states occur. A positive proximity tail causes a shift in the resonance energies to the right of the zero-tail spectrum shown in figure 5, whereas a negative proximity tail shifts the spectrum to the left. These results suggest a novel method for detecting the sign of the electron–electron interaction in the N metal: on suppressing the order parameter in the normal region, the resonances will shift either to higher energies, indicating a repulsive interaction, or to lower energies, in the case of an attractive interaction. This suppression can be achieved by, for example, applying a magnetic field or via a control current in the superconductor.

This work was funded by the EPSRC and the EU TMR programme.

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